Fully differential VBF Higgs production at NNLO

Matteo Cacciari,[1, 2, 3, 4] Frédéric A. Dreyer,[2, 3, 4] Alexander Karlberg,[5] Gavin P. Salam,[4, *] and Giulia Zanderighi[4, 5]

[1]*Université Paris Diderot, Paris, France*
[2]*Sorbonne Universités, UPMC Univ Paris 06, UMR 7589, LPTHE, F-75005, Paris, France*
[3]*CNRS, UMR 7589, LPTHE, F-75005, Paris, France*
[4]*CERN, PH-TH, CH-1211 Geneva 23, Switzerland*
[5]*Rudolf Peierls Centre for Theoretical Physics, 1 Keble Road, University of Oxford, UK*

We calculate the fully differential next-to-next-to-leading-order (NNLO) corrections to vector-boson fusion (VBF) Higgs production at proton colliders, in the limit in which there is no cross-talk between the hadronic systems associated with the two protons. We achieve this using a new "projection-to-Born" method that combines an inclusive NNLO calculation in the structure-function approach and a suitably factorised next-to-leading-order (NLO) VBF Higgs plus 3-jet calculation, using appropriate Higgs plus 2-parton counterevents. An earlier calculation of the fully inclusive cross section had found small NNLO corrections, at the 1% level. In contrast, the cross section after typical experimental VBF cuts receives NNLO contributions of about 5-6%, while differential distributions show corrections of up to 10-12% for some standard observables. The corrections are often outside the NLO scale-uncertainty band.

Following the discovery in 2012 of the Higgs boson [1, 2], one of the main tasks for particle physics today is the accurate determination of its properties and couplings. For the coming decade at least, these studies will take place at CERN's Large Hadron Collider (LHC).

The most relevant production channels for the Higgs boson at the LHC are gluon fusion (ggH), vector-boson fusion (VBFH), production in association with a vector boson (VH) and with a top-quark pair (ttH) [3]. VBFH is special for a number of reasons [4, 5]: it has the largest cross section of the processes that involves tree-level production of the Higgs boson (and is second largest among all processes); it has a distinctive signature of two forward jets, which makes it possible to tag the events and so identify Higgs decays that normally have large backgrounds, e.g. $H \to \tau^+\tau^-$; the Higgs transverse momentum is non-zero even at lowest order, which facilitates searches for invisible decay modes [6, 7]; and it also brings particular sensitivity to the charge-parity properties of the Higgs boson, and non-standard Higgs interactions, through the angular correlations of the forward jets [8].

Given the key role of VBF Higgs-boson production at the LHC, it is of paramount importance to have a precise prediction for its production. The total cross section was calculated to NNLO in Refs. [9, 10] using the structure-function approach [11], showing small corrections relative to the NLO and tiny renormalisation and factorisation scale uncertainties, well below 1%. However experimental measurements are necessarily restricted to a subset of phase space. In particular, because of their use of transverse-momentum cuts on the forward tagging jets, one might imagine that there are important NNLO corrections, associated with those jet cuts, that would not be seen in a fully inclusive calculation. Currently, the fully differential VBFH cross section is known only

to NLO [12]. It appears to have small scale uncertainties.

One can think of the VBFH process, represented at the Born level in Fig. 1a, as involving two Deeply Inelastic Scatterings (DIS), one for each of the incoming protons. Each DIS process produces a vector boson, $W^\pm$ or Z, and the fusion of the vector bosons produces a Higgs boson. The structure-function approach [11] used for the NNLO total cross section [9, 10] assumes that the upper and lower hadronic sectors factorise from each other, i.e. that there is no cross-talk between them. Factorisation is believed to be accurate to better than 1% in the experimentally relevant kinematic region [10, 13–15].[1]

The reason that the structure function approach does not provide a fully differential cross section is related to the fact that the DIS coefficient functions used in the calculation implicitly integrate over hadronic final states. In this letter, we introduce a new "projection-to-Born" approach to eliminate this limitation, and thus provide the first fully differential NNLO calculation of VBF Higgs production in the factorised approximation.

Let us start by recalling that the cross section in the structure-function approach is expressed [9–11] as a sum of terms involving products of structure functions, e.g. $F_2(x_1, Q_1^2)F_2(x_2, Q_2^2)$, where $Q_i^2 = -q_i^2 > 0$ is given in terms of the 4-momentum q_i of the (outgoing) exchanged vector boson i. The x_i values are fixed by the relation $x_i = -Q_i^2/(2P_i.q_i)$, where P_i is the momentum of proton i. To obtain the total cross section, one integrates over all q_1, q_2 that can lead to the production of a Higgs boson. If the underlying upper (lower) scattering is Born-like,

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*On leave from CNRS, UMR 7589, LPTHE, F-75005, Paris, France

[1] The factorisation of the two sectors is exact if one imagines two copies of QCD, QCD_1 and QCD_2, respectively for the upper and lower sectors, where each of the two QCD copies interacts with the electroweak (EW) sector, but not with the other QCD copy. This observation could be exploited, for example, in automated calculations and for determining corrections beyond the factorised approximation.
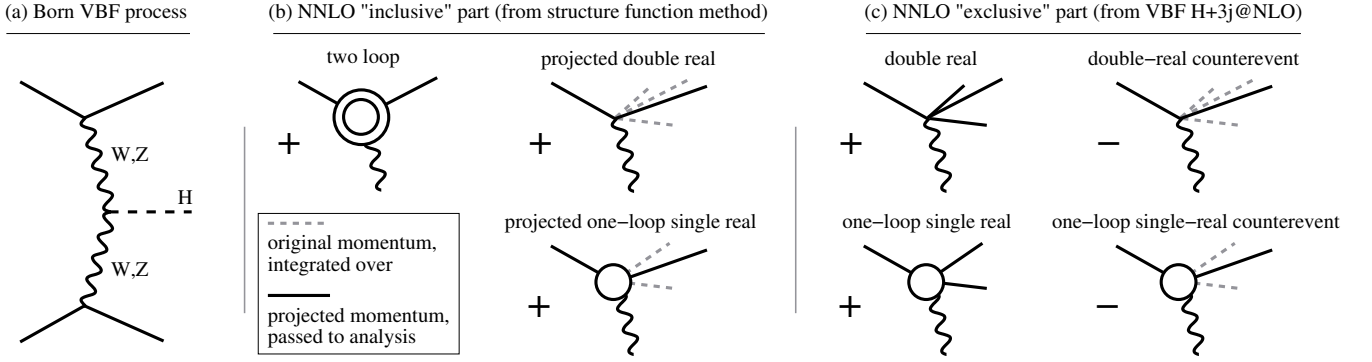
FIG. 1: (a) Illustration of the Born VBFH process. (b) NNLO corrections to the upper sector of the VBF process, from the "inclusive" part of our calculation. (c) Corresponding "exclusive" part. The double-real and one-loop single-real counterevents in the exclusive part cancel the projected double-real and one-loop single-real contributions in the inclusive part. In the "projected" and "counterevent" contributions, the dashed lines corresponds to the full set of parton momenta that are integrated over (for the structure functions, this integral is implicit in the derivation of the coefficient functions), while the solid lines correspond to the partons that are left over after projection to Born-like kinematics and then passed to the analysis. The projection does not change the direction of initial partons and so the corresponding incoming dashed lines are implicit.

quark $\to$ quark $+ V$, then it is straightforward to show that knowledge of the vector-boson momentum q_1 (q_2) uniquely determines the momenta of both the incoming and outgoing (on-shell) quarks,

$$p_{\text{in},i} = x_i P_i, \qquad p_{\text{out},i} = x_i P_i - q_i. \qquad (1)$$

We exploit this feature in order to assemble a full calculation from two separate ingredients. For the first one, the "inclusive" ingredient, we remain within the structure function approach, and for each set of q_1 and q_2 use Eq. (1) to assign VBF Born-like kinematics to the upper and lower sectors. This is represented in Fig. 1b (showing, for brevity, just the upper sector): for the two-loop contribution, the Born kinematics that we assign corresponds to that of the actual diagrams; for the tree-level double-real and one-loop single-real diagrams, it corresponds to a projection from the true kinematics ($2 \to H + n$ for $n = 3, 4$) down to the Born kinematics ($2 \to H + 2$). The projected momenta are used to obtain the "inclusive" contribution to differential cross sections. Note that the Higgs momentum is unaffected by the projection.

Our second, "exclusive", ingredient starts from the NLO fully differential calculation of vector-boson fusion Higgs production with three jets [16, 17], as obtained in a factorised approximation, i.e. where there is no crosstalk between upper and lower sectors.[2] Thus each parton can be uniquely assigned to one of the upper or lower sectors and the two vector-boson momenta can be unambiguously determined. For each event in a Monte Carlo integration over phase space, with weight w, we add a

counterevent, with weight $-w$, to which we assign projected Born VBF kinematics based on the vector-boson momenta and Eq. (1). This is illustrated in Fig. 1c. From the original events, we thus obtain the full momentum structure for tree-level double-real and one-loop single-real contributions. Meanwhile, after integration over phase space, the counterevents exactly cancel the projected tree-level double-real and one-loop single-real contributions from the inclusive part of the calculation. Thus the sum of the inclusive and exclusive parts gives the complete differential NNLO VBFH result.[3]

For the implementation of the inclusive part of the calculation, we have taken the phase space from POWHEG's Higgs plus two-jet VBF calculation [20], while the matrix element has been coded with structure functions evaluated using parametrised versions [21, 22] of the NNLO DIS coefficient functions [23–25] integrated with HOPPET v1.1.5 [26]. We have tested our implementation against the results of one of the codes used in Ref. [9, 10] and found agreement, both for the structure functions and the final cross sections. We have also checked that switching to the exact DIS coefficient functions has a negligible impact. A further successful comparison of the evaluation of structure functions was made against APFEL v.2.4.1[27].

For the exclusive part of the calculation, as a starting point we took the NLO (i.e. fixed-order, but not parton-shower) part of the POWHEG H+3-jet VBF code [17], itself based on the calculation of Ref. [16], with tree-level matrix elements from MadGraph 4 [28]. This code already uses a factorised approximation for the matrix element, however for a given phase-space point it sums over

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[2] The NLO calculation without this approximation is given in Ref. [18].

[3] Our approach can be contrasted with the differential NNLO structure-function type calculation for single-top production [19] in that we do not need any fully differential ingredients at NNLO.

matrix-element weights for the assignments of partons to upper and lower sectors. We therefore re-engineered the code so that for each set of 4-momenta, weights are decomposed into the contributions for each of the different possible sets of assignments of partons to the two sectors. For every element of this decomposition it is then possible to unambiguously obtain the vector-boson momenta and so correctly generate a counterevent. The `POWHEG-BOX`'s [29, 30] "tagging" facility was particularly useful in this respect, notably for the NLO subtraction terms. To check the correctness of the assignment to sectors, we verified that as the rapidity separation between the two leading jets increases, there was a decreasing relative fraction of the cross section for which partons assigned to the upper (lower) sector were found in the rapidity region associated with the lower (upper) leading jet. We also tested that the sum of inclusive and exclusive contributions at NLO agrees with the `POWHEG` NLO implementation of the VBF $H+2$-jet process.

To investigate the phenomenological consequences of the NNLO corrections, we study 13 TeV proton-proton collisions. We use a diagonal CKM matrix, full Breit-Wigners for the W, Z and the narrow-width approximation for the Higgs boson. We take NNPDF 3.0 parton distribution functions at NNLO with $\alpha_s(M_Z) = 0.118$ (`NNPDF30_nnlo_as_0118`) [31], also for our LO and NLO results. We have five light flavours and ignore contributions with top-quarks in the final state or internal lines. We set the Higgs mass to $M_H = 125$ GeV, compatible with the experimentally measured value [32]. Electroweak parameters are set according to known experimental values and tree-level electroweak relations. As inputs we use $M_W = 80.398$ GeV, $M_Z = 91.1876$ GeV and $G_F = 1.16637 \times 10^{-5}$ GeV^{-1}. For the widths of the vector bosons we use $\Gamma_W = 2.141$ GeV and $\Gamma_Z = 2.4952$ GeV.

Some care is needed with the renormalisation and factorisation scale choice. A natural option would be to use Q_1 and Q_2 as our central values for the upper and lower sectors, respectively. While this is straightforward in the inclusive code, in the exclusive code we had the limitation that the underlying `POWHEG-BOX` code can presently only easily assign a single scale (or set of scales) to a given event. However, for each `POWHEG` phase-space point, we have multiple upper/lower classifications of the partons, leading to several $\{Q_1, Q_2\}$ pairs for each event. Thus the use of Q_1 and Q_2 would require some further degree of modification of the `POWHEG-BOX`, which we leave to future work. We instead choose a central scale that depends on the Higgs transverse momentum $p_{t,H}$:

$$\mu_0^2(p_{t,H}) = \frac{M_H}{2}\sqrt{\left(\frac{M_H}{2}\right)^2 + p_{t,H}^2}\,. \tag{2}$$

This choice of μ_0 is usually close to $\sqrt{Q_1 Q_2}$. It represents a good compromise between satisfying the requirement of a single scale for each event, while dynamically adapting to the structure of the event. In order to estimate missing

	$\sigma^{\text{(no cuts)}}$ [pb]	$\sigma^{\text{(VBF cuts)}}$ [pb]
LO	$4.032^{+0.057}_{-0.069}$	$0.957^{+0.066}_{-0.059}$
NLO	$3.929^{+0.024}_{-0.023}$	$0.876^{+0.008}_{-0.018}$
NNLO	$3.888^{+0.016}_{-0.012}$	$0.826^{+0.013}_{-0.014}$

TABLE I: Cross sections at LO, NLO and NNLO for VBF Higgs production, fully inclusively and with VBF cuts. The quoted uncertainties correspond to scale dependence, while statistical errors at NNLO are about 0.1% with VBF cuts and much smaller without.

higher-order uncertainties, we vary the renormalisation and factorisation scales symmetrically (i.e. keeping $\mu_R = \mu_F$) by a factor 2 up and down around μ_0.[4]

To pass our VBF selection cuts, events should have at least two jets with transverse momentum $p_t > 25$ GeV; the two hardest (i.e. highest p_t) jets should have absolute rapidity $|y| < 4.5$, be separated by a rapidity $\Delta y_{j_1,j_2} > 4.5$, have a dijet invariant mass $m_{j_1,j_2} > 600$ GeV and be in opposite hemispheres ($y_{j_1}y_{j_2} < 0$). Jets are defined using the anti-k_t algorithm [33], as implemented in `FastJet v3.1.2` [34], with radius parameter $R = 0.4$.

Results are shown in table I for the fully inclusive cross section and with our VBF cuts. One sees that the NNLO corrections modify the fully inclusive cross section only at the percent level, which is compatible with the findings of Ref. [9]. However, after VBF cuts, the NNLO corrections are about 5 times larger, reducing the cross section by $5-6\%$ relative to NLO. The magnitude of the NNLO effects after cuts implies that it will be essential to take them into account for future precision studies. Note that in both the inclusive and VBF-cut cases, the NNLO contributions are larger than would be expected from NLO scale variation.

Differential cross sections are shown in Fig. 2, for events that pass the VBF cuts. From left to right, the plot shows the transverse momentum distributions for the two leading jets, p_{t,j_1} and p_{t,j_2}, for the Higgs boson, $p_{t,H}$, and the distribution for the rapidity separation between the two leading jets, $\Delta y_{j_1,j_2}$. The bands and the patterned boxes denote the scale uncertainties, while the vertical error-bars denote the statistical uncertainty. The effect of the NNLO corrections on the jets appears to be to reduce their transverse momentum, leading to negative (positive) corrections in regions of falling (rising) jet spectra. One can see effects of up to $10-12\%$. Turning to $p_{t,H}$, one might initially be surprised that such an inclusive observable should also have substantial NNLO corrections, of about 8% for low and moderate $p_{t,H}$. Our

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[4] We verified that an expanded scale variation, allowing $\mu_R \neq \mu_F$ with $\frac{1}{2} < \mu_R/\mu_F < 2$, led only to very small changes in the NNLO scale uncertainties for the VBF-cut cross section and the $p_{t,H}$ distribution.
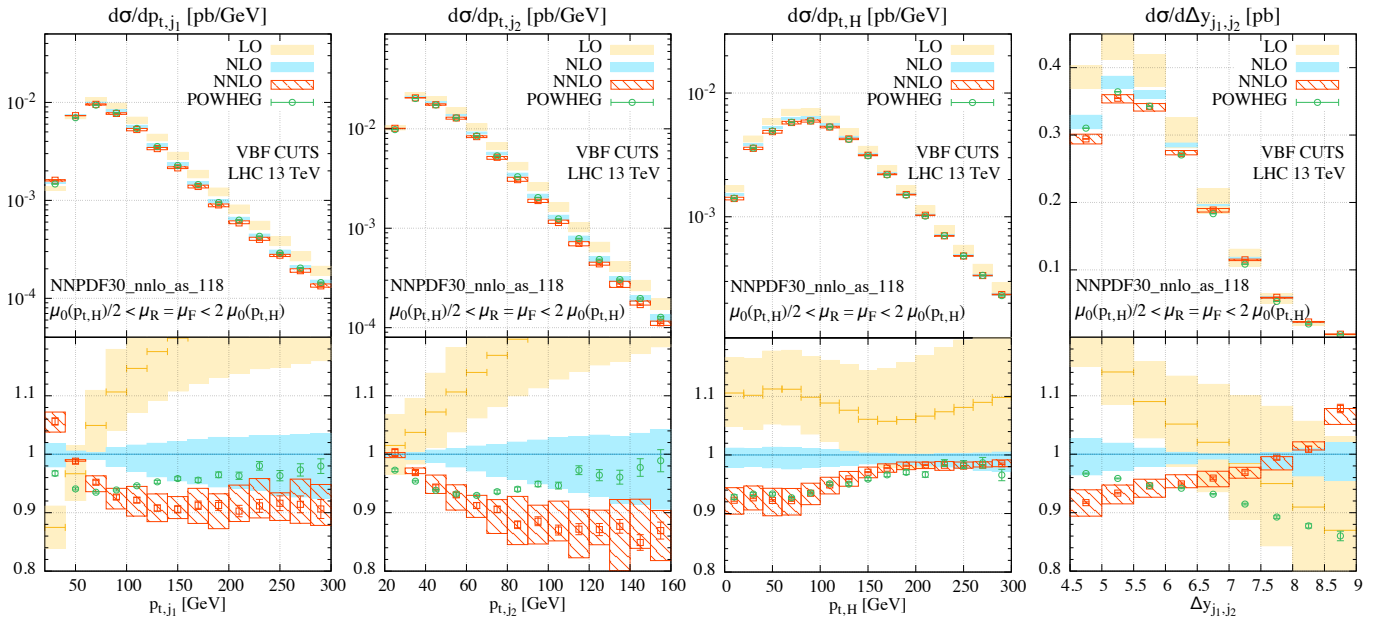
FIG. 2: From left to right, differential cross sections for the transverse momentum distributions for the two leading jets, p_{t,j_1} and p_{t,j_2}, for the Higgs boson, $p_{t,H}$, and the distribution for the rapidity separation between the two leading jets, $\Delta y_{j_1,j_2}$.

interpretation is that since NNLO effects redistribute jets from higher to lower p_t's (cf. the plots for p_{t,j_1} and p_{t,j_2}), they reduce the cross section for any observable defined with VBF cuts. As $p_{t,H}$ grows larger, the forward jets tend naturally to get harder and so automatically pass the p_t thresholds, reducing the impact of NNLO terms.

As observed above for the total cross section with VBF cuts, the NNLO differential corrections are sizeable and often outside the uncertainty band suggested by NLO scale variation. One reason for this might be that NLO is the first order where the non-inclusiveness of the jet definition matters, e.g. radiation outside the cone modifies the cross section. Thus NLO is, in effect, a leading-order calculation for the exclusive corrections, with all associated limitations.

To further understand the size of the NNLO corrections, it is instructive to examine a NLO plus parton shower (NLOPS) calculation, since the parton shower will include some approximation of the NNLO corrections. For this purpose we have used the POWHEG VBF H+2-jet calculation [20], showered with PYTHIA version 6.428 with the Perugia 2012 tune [35]. The POWHEG part of this NLOPS calculation uses the same PDF, scale choices and electroweak parameters as our full NNLO calculation. The NLOPS results are included in Fig. 2, at parton level, with multi-parton interactions (MPI) switched off. They differ from the NLO by an amount that is of a similar order of magnitude to the NNLO effects. This lends support to our interpretation that final (and initial)-state radiation from the hard partons is responsible for a substantial part of the NNLO corrections. However, while the NLOPS calculation reproduces the shape of the NNLO corrections for some observables

(especially $p_{t,H}$), there are others for which this is not the case, the most striking being perhaps $\Delta y_{j_1,j_2}$. Parton shower effects were also studied in Ref. [36], using the MC@NLO approach [37]. Various parton showers differed there by up to about 10%.

In addition to the NNLO contributions, precise phenomenological studies require the inclusion of EW contributions and non-perturbative hadronisation and MPI corrections. The former are of the same order of magnitude as our NNLO corrections [13]. Using Pythia 6.428 and Pythia 8.185 we find that hadronisation corrections are between -2 and 0%, while MPI brings up to $+5\%$ at low p_t's. The small hadronisation corrections appear to be due to a partial cancellation between shifts in p_t and rapidity. We leave a combined study of all effects to future work. The code for our calculation will also be made public.

With the calculation presented in this letter, differential VBF Higgs production has been brought to the same NNLO level of accuracy that has been available for some time now for the ggH [38, 39] and VH [40] production channels. This constitutes the first fully differential NNLO $2 \rightarrow 3$ hadron-collider calculation, an advance made possible thanks to the factorisable nature of the process. The NNLO corrections are non-negligible, 5–10%, i.e. an order of magnitude larger than the corrections to the inclusive cross section. Their size might even motivate a calculation one order higher, to N^3LO, to match the precision achieved recently for the ggH total cross section [41]. With the new "projection-to-Born" approach introduced here, we believe that this is within reach. It would also be of interest to obtain NNLO plus parton shower predictions, again matching the accuracy

achieved recently in ggH [42, 43].

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